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Economics Series

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Abstract

We consider a Diamond–type model of endogenous growth in which there are three assets: outside money, government bonds, and equity. Due to productivity shocks, the equity return is uncertain, and risk averse investors require a positive equity premium. Typically, there exist two steady states, but only one of them is stable, both in the forward perfect foresight dynamics and under adaptive expectations. Tight monetary policy is harmful for growth in the stable steady state. These results hold under four different monetary policy strategies applied by the monetary authority. A monetary contraction increases the bond return, reduces the equity premium and thereby capital investment and growth.

Keywords

Monetary policy, endogenous growth, equity premium

JEL Classifications

D84, E52, O42

Comments
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Contents

1 Introduction

This paper deals with the question of how monetary policy affects growth. The traditional literature on monetary growth theory emphasizes the Mundell–Tobin or "portfolio" effect which says that money growth affects the capital stock positively, since higher inflation reduces the return on real balances which induces investors to reallocate savings from money to capital (see Mundell (1965), Tobin (1965)). Within dynamic general equilibrium models, however, such an effect is hard to find and most studies report either superneutrality of money or even a negative relation between money growth and real activity.[1] As the theoretical literature, also empirical studies on this issue draw different conclusions.[2]

Most of the theoretical literature considers only a single outside asset (money) and examines the effects of variations of the growth rate of this asset. In such a framework, however, the impact of different monetary strategies on real activity cannot be studied adequately. To address this issue, Schreft and Smith (1997, 1998) consider a Diamond–type overlapping generations model with outside money and government bonds in which different monetary policy strategies like a constant money growth rule, an inflation targeting or an interest targeting rule can be studied. They show that there exist multiple steady states and that the effects of monetary policy on the output level in these steady states are ambiguous.

Schreft and Smith assume that government bonds and physical capital are perfect substitutes in the portfolios of consumers and that firms finance their capital investments by loans for which they pay the same interest rate as the government on treasury bills. Thus, the rates of return on government bonds and capital coincide, and monetary policy affects both interest rates in the same way. For instance, if a higher bond return is induced by a tightening of monetary policy, the capital return and thereby capital investment increase as well. However, this assumption of Diamond–type growth models neglects that firms finance (part of) their capital investment by equity and that the equity return exceeds the return on government bonds. If there is a positive spread between the equity and the bond return, a higher bond return need not increase the capital return, but may decrease the risk premium, induce investors to buy less equity, and thereby induce firms to accumulate less capital. Hence, the traditional Mundell–Tobin

[1]See, for instance, Sidrauski (1967), Brock (1974) and Stockman (1981). More recently, Jones and Manuelli (1995) show a negative correlation between inflation and growth in an endogenous growth model, and Azariadis and Smith (1996) find a negative relationship between inflation and output at high inflation which is reversed at low inflation. For surveys on money and growth, see Orphanides and Solow (1990) and von Thadden (1999).

[2]In fact, most empirical studies report a negative correlation between inflation and growth, which is particularly strong at high inflation rates. However, Bullard and Keating (1995) and McCandless and Weber (1995) show that inflation (money growth resp.) and growth are uncorrelated in large samples, while they are positively correlated in subsamples of low–inflation countries.

effect reappears.

This paper departs from the model of Schreft and Smith in two important ways. First, firms finance capital investments by equity instead of bonds and the equity return exceeds the bond return, since there are stochastic productivity shocks and since consumers are risk averse. Second, because of an Arrow–Romer spillover of capital investment on labor productivity, the aggregate technology exhibits increasing returns to scale which gives rise to endogenous growth. This enables us to study the growth effects of monetary policy.

Specifically, consumers transfer the labor income of their first lifetime period to the second period by means of three assets: money, government bonds, and equity. Because of a cash–in–advance constraint consumers hold money even if it is return dominated by bonds and equity.[3] Since consumers are risk averse and since the equity return is uncertain, both the equity and the bond demand can be positive when there is a positive (expected) equity premium. Firms finance capital investments only by issuing equity. The government consumes a fixed share of output and finances its deficit by bonds and by seignorage, whereas the monetary authority controls the money supply by conducting open market operations. Hence, the monetary authority determines the seignorage revenue of the government and can apply different types of monetary strategies. We consider four monetary policy strategies: a constant money growth rule, a stabilization of the ratio of money to bonds, an inflation targeting and an interest rate targeting rule.

As the model of Schreft and Smith, our model may well have multiple steady states[4], depending on the parameter specifications and on the monetary strategy. However, only one of these steady states is locally stable, both in the forward perfect foresight dynamics and under adaptive expectations. Moreover, we find that money affects growth positively in any stable steady state and for any type of monetary strategy. Only under interest rate targeting the growth effect is ambiguous and depends on the size of the risk premium. If the risk premium is too low, an increase in the nominal interest target is accompanied by a larger increase in inflation which leads to a lower real interest rate and thus to higher growth.

A loose monetary policy raises the seignorage revenue which allows the government to issue less bonds and which reduces thereby the real bond return. Since the expected capital return is constant in our simple AK–type growth model, this raises the equity premium and induces consumers to shift more savings to capital which leads to a higher growth rate. By the same mechanism, an increase of fiscal expenditures unambiguously reduces the growth rate in any stable steady

[3]In Schreft and Smith's model the use of money is motivated by random liquidity shocks and liquidity is provided by banks. Our cash-in-advance constraint is equivalent to the banks' liquidity constraint of Schreft and Smith (1997) if consumers are assumed to have logarithmic preferences.

[4]We use the term steady state to denote a balanced growth path.

state, which is in accordance with the findings of Barro (1990), since government services do not affect production or utility in our model.

Our argument provides an alternative to related work of van der Ploeg and Alogoskoufis (1994) who consider the impact of monetary policy in an overlapping generations model in the spirit of Weil (1991) and with endogenous growth due to an Arrow–Romer externality. They find that higher money growth rates affect growth positively since currently living generations do not benefit from tax cuts in the future, and therefore consume less and invest more which increases the long–run real growth rate. In our model, however, such effects of intertemporal taxation are absent, and growth is raised by the monetary policy's impact on asset prices and the equity premium.

Our model also relates to Hahn and Solow (1995, Chapter 2) who consider a Diamond–type growth model in which consumers hold money because of a cash–in–advance constraint. Unlike Schreft and Smith, Hahn and Solow not only focus on steady states in which money is return dominated by bonds and in which the cash–in–advance constraint on consumers is binding (consumers are *liquidity constrained*), but they also analyse steady states in which the rate of returns on money and bonds are equal (the nominal interest rate is zero) and in which consumers are *portfolio indifferent.* In this paper, we also consider such portfolio–indifference steady states, but we find that their existence depends crucially on the monetary strategy. For instance, under constant money growth or under inflation targeting, portfolio indifference steady states exist only in pathological cases, and under a fixed money–bond ratio they only exist if the monetary policy is sufficiently loose. When they exist, however, monetary policy has no effect on the growth rate in these steady states.

The remainder of the paper is organized as follows. The next section describes the economic agents and derives the model's equilibrium conditions. Section 3 examines the perfect foresight equilibrium growth paths of the model and discusses the existence, multiplicity and comparative statics of steady states under four different monetary strategies. Section 4 looks at the dynamics with adaptive expectations and shows that the stability features of the forward perfect foresight dynamics are preserved. Section 5 concludes.

2 The Model

Consider an overlapping generations model in which there are three types of agents: consumers, firms and a government. They trade a composite consumption good and labor as well as three types of financial assets: money, bonds and equity. The government issues fiat money and bonds, while firms finance their capital investments by issuing equity. Since the capital return is uncertain due to stochastic productivity shocks, risk averse consumers require a positive equity premium. Furthermore, because of a liquidity constraint money is held even when it is return dominated by bonds. In detail the agents are described as follows.

2.1 The Consumer

There is a single representative consumer who is endowed with one unit of labor in his first period which he supplies inelastically, whereas he consumes in his second period of life only. He aims to transfer his real labor income w_t to the second period by holding money (m_t), bonds (b_t) or equity (e_t). The corresponding real gross rates of return are $R_t^M = p_t/p_{t+1}$, R_t^B and R_t^E, respectively. The consumer faces a liquidity constraint for money holdings, $m_t \geq \lambda w_t$, where $\lambda \in [0, 1)$. While R_t^M and R_t^B are foreseen with certainty, the equity return is uncertain and the consumer expects rationally that R_t^E is normally distributed with density function $g(\cdot; \overline{R}_t^E, \sigma^2)$, where $\overline{R}_t^E$ is the mean and σ^2 the variance. Moreover, the consumer is risk averse and his von Neumann–Morgenstern utility function is assumed to be $u_t(c_{t+1}) = -e^{-\rho_t c_{t+1}}$ where $\rho_t = \rho/w_t$ and $\rho > 0$ is given. This means that the consumer's absolute risk aversion, ρ_t, decreases as his income increases and will imply that, on a balanced growth path, he has constant relative risk aversion.[5] The consumer's decision problem is

$$\max_{c_{t+1}, m_t, b_t, e_t} Eu_t(c_{t+1}) \tag{1}$$

$$\text{s.t. } c_{t+1} \leq R_t^M m_t + R_t^B b_t + R_t^E e_t,\ m_t + b_t + e_t \leq w_t,\ m_t \geq \lambda w_t,\ e_t \geq 0.$$

Notice that a necessary condition for a solution to this problem is that $R_t^B \geq R_t^M$, since we have not imposed a lower bound on bond holdings. If $R_t^B < R_t^M$, the consumer could issue arbitrarily many bonds and hold cash in order to guarantee an arbitrarily high consumption level. When $R_t^B > R_t^M$, the liquidity constraint must be binding, hence we have

$$R_t^B \geq R_t^M,\ m_t \geq \lambda w_t,\ \left(R_t^B - R_t^M\right)(m_t - \lambda w_t) = 0\ . \tag{2}$$

The following Lemma shows that the equity demand is an increasing function of income and the expected equity premium.

Lemma 1 *The consumer's equity demand is* $e_t^d = \max\left\{\dfrac{\overline{R}_t^E - R_t^B}{\rho\sigma^2} w_t, 0\right\}$.

[5] It may appear a peculiar feature of the utility function that it is decreasing in income w_t. Note, however, that w_t is not a choice variable for the consumer and, moreover, that this is only a "cardinal" aspect of the utility function in that it could be easily overcome by assuming for example $u_t(c_{t+1}) = -\rho_t^\beta e^{-\rho_t c_{t+1}}$, with β sufficiently large, without changing anything in the consumer's preference structure and in the analytical results. The alternative to assume a CRRA utility function $c_{t+1}^{1-\rho}/(1-\rho)$ together with a lognormal density for the equity return would not work in the present context since c_{t+1} is a sum $a + bR_t^E$ with R_t^E the variable of integration in the integral that represents the expected utility. Then the integral could not be solved explicitly.

Proof: The consumer's problem is

$$\max_{m_t,b_t,e_t} \int -e^{-\rho_t\left[R_t^M m_t + R_t^B b_t + re_t\right]} g\left(r; \overline{R}_t^E, \sigma^2\right) dr \ ,$$

subject to the constraints in (1). The integral can be written

$$-e^{-\rho_t\left[R_t^M m_t + R_t^B b_t\right]} \int e^{-\rho_t re_t} g\left(r; \overline{R}_t^E, \sigma^2\right) dr$$

and, using the formula $\int e^{tx} g\left(x; \nu, \sigma^2\right) dx = e^{t\nu + \left(t^2/2\right)\sigma^2}$, it becomes

$$-e^{-\rho_t\left[R_t^M m_t + R_t^B b_t + \overline{R}_t^E e_t - (\rho_t/2) e_t^2 \sigma^2\right]}.$$

Substituting $b_t = w_t - m_t - e_t$, the problem is equivalent to

$$\max_{m_t \geq \lambda w_t, e_t \geq 0} \left(R_t^M - R_t^B\right) m_t + \left(\overline{R}_t^E - R_t^B\right) e_t - (\rho_t/2)\, e_t^2 \sigma^2.$$

The solution of this problem leads immediately to (2) and to the claimed equity demand. □

An alternative formulation of Lemma 2.1 is

$$\overline{R}_t^E = R_t^B + \frac{\rho \sigma^2}{w_t} e_t^d \tag{3}$$

whenever $e_t^d > 0$. This reflects the equity premium required due to uncertainty and risk aversion of consumers.

2.2 The Firms

Firms are risk–neutral and they produce output Y_t from labor input L_t and capital input K_t using the production technology $Y_t = \Phi_t F(K_t, A_t L_t)$. Φ_t is a total factor productivity shock which is realized only after capital is installed and workers are hired, and all Φ_t are independently and normally distributed with mean 1 and variance σ_Φ^2.[6] F exhibits constant returns to scale and A_t measures labor productivity at time t. Firms have to install capital a period in advance by issuing equity. Thus equity supply in period $t-1$ is $e_{t-1} = K_t$. Since firms finance capital input by issuing equity instead of bonds, firms maximize profit with respect to labor while capital demand is determined consistently with capital supply of the investors and with the rationality of their expectations (see Hahn and Solow (1995, Chapter 4)).

[6]This specification implies that output and the equity return can be negative with positive probability. However, these pathological realizations occur only with a small probability if σ_Φ is not too large.

Given K_t at the beginning of period t, the firm's expected profit maximization problem is

$$\max_{L_t} E\left(\Phi_t F(K_t, A_t L_t) - w_t L_t\right)$$

which leads to

$$w_t = A_t\left(f(k_t) - k_t f'(k_t)\right) \ , \tag{4}$$

where $k_t = K_t/(A_t L_t)$ and $f(k_t) = F(k_t, 1)$. To endogenize A_t, we assume a positive spillover from aggregate investment on labor productivity, as suggested by Arrow (1962) and Romer (1986). To be consistent with long–run endogenous growth, we use a linear relationship of the form

$$A_t = \frac{1}{a}\overline{K}_t \ , \tag{5}$$

where $\overline{K}_t$ is the aggregate capital stock. The size of firms is normalized to 1, so $\overline{K}_t = K_t$ has to hold in equilibrium, and labor market clearing implies $L_t = 1$. This together with (5) substituted in (4) implies

$$w_t = \alpha K_t = \alpha e_{t-1} \quad \text{where} \quad \alpha = \frac{f(a)}{a} - f'(a) \ . \tag{6}$$

This in turn implies that the equity return is

$$R^E_{t-1} = \frac{\Phi_t F(K_t, A_t L_t) - w_t L_t}{K_t} = \Phi_t \frac{f(a)}{a} - \alpha \ .$$

Since Φ_t is normally distributed, R^E_{t-1} is also normally distributed. Rational expectations of investors imply that

$$\overline{R}^E_{t-1} = \frac{f(a)}{a} - \alpha = f'(a) \quad \text{and} \quad \sigma^2 = \sigma^2_\Phi \left(\frac{f(a)}{a}\right)^2 \ . \tag{7}$$

Thus, the expected equity return is constant over time and equals the marginal product of capital at the balanced growth level of the capital intensity. This result is due to the constant returns assumption and it implies that if there was no uncertainty and if firms could issue bonds instead of equity, the firm would choose the same level of capital input at the interest rate $R^B_t = \overline{R}^E_t$. Therefore bonds and equity would then also be equivalent from the perspective of firms.

2.3 The Government

The government spends an amount g_t of the composite consumption good and finances its deficit by issuing bonds and money. We assume that the government's expenditures are a constant share of expected output $EY_t = F(K_t, A_t L_t) = (f(a)/a)\, K_t$, i.e. $g_t = q\frac{f(a)}{a}e_{t-1}$ where $q \in [0, 1)$. The government has to satisfy its budget constraint

$$R^B_{t-1}b_{t-1} + q\frac{f(a)}{a}e_{t-1} = b_t + m_t - R^M_{t-1}m_{t-1}\ . \tag{8}$$

The left hand side denotes real government expenditures on consumption and interest payments, and the right hand side contains the newly issued bonds and a seignorage term.

2.4 The Equilibrium

By Walras's law, equilibrium on the labor, money, bond, and equity markets imply that also the goods market is in equilibrium, i.e. $Y_t = c_t + g_t + e_t$. Notice that only consumption adjusts to productivity shocks, since government consumption is predetermined and since investment (=equity demand) is a constant fraction of labour income which is not affected by productivity shocks (see (6)). Substituting (6) into the consumer's budget constraint yields

$$m_t + e_t + b_t = \alpha e_{t-1}\ . \tag{9}$$

Moreover, inserting (6) and (7) into (3) implies

$$R^B_t = f'(a) - \frac{\rho\sigma^2}{\alpha}\frac{e_t}{e_{t-1}}\ . \tag{10}$$

Using (6) again, (2) becomes

$$R^B_t \geq R^M_t,\ m_t \geq \lambda\alpha e_{t-1},\ \left(R^B_t - R^M_t\right)(m_t - \lambda\alpha e_{t-1}) = 0\ . \tag{11}$$

Equations (8), (9), (10) and (11) are dynamical equations with endogenous variables m_t, e_t, b_t, R^B_t and R^M_t that permit to study the evolution of the system provided the number of state variables is reduced to four. This will be achieved by specifications of monetary policy rules that we study in the sequel.

3 Monetary Policy

We will consider in this section four different monetary policy rules: a constant money growth policy, a policy in which the central bank stabilizes the ratio of money to government bonds, an interest rate targeting and an inflation targeting policy.

3.1 Constant Money Growth

Adopting a constant money growth rule means $M_t = \mu M_{t-1}$ for all t, with $\mu \geq 1$, where M_t is nominal money. Recalling that $R^M_t = p_t/p_{t+1}$, this can equivalently be written

$$m_t = \mu R^M_{t-1}m_{t-1}\ . \tag{12}$$

We will first consider the situation in which consumers are liquidity constrained, i.e. in which the nominal interest rate is positive and the liquidity constraint binds. That is, we suppose $m_t = \lambda\alpha e_{t-1}$ and $R_t^B > R_t^M$, and we will show that there are in general two or no steady states with this feature. Later on, we consider the case in which the nominal interest rate is zero and in which consumers are portfolio indifferent, but we will show that there exists generically no such a steady state.

Inserting (10) and (12) into the government's budget constraint (8) and using $m_t = \lambda\alpha e_{t-1}$ we obtain

$$\left(f'(a) - \frac{\rho\sigma^2}{\alpha}\frac{e_{t-1}}{e_{t-2}}\right) b_{t-1} + q\frac{f(a)}{a}e_{t-1} = b_t + \left(1 - \frac{1}{\mu}\right)\lambda\alpha e_{t-1} \ . \tag{13}$$

Similarly, (9) becomes

$$e_t + b_t = (1-\lambda)\,\alpha e_{t-1} \ . \tag{14}$$

Equations (13) and (14) constitute the dynamical system to be studied, provided $R_t^M < R_t^B$. Since these equations are linearly homogenous, they can be reduced to a one-dimensional equation. Using $\gamma_t = e_t/e_{t-1}$ for the growth rate and $x_t = b_t/w_t = b_t/\left(\alpha e_{t-1}\right)$ for the share of bonds in income, (14) becomes

$$\gamma_t = \alpha\left(1 - \lambda - x_t\right) \ , \tag{15}$$

while (13) can be rewritten as

$$\left(f'(a) - \frac{\rho\sigma^2}{\alpha}\gamma_{t-1}\right)\frac{\alpha x_{t-1}}{\gamma_{t-1}} + q\frac{f(a)}{a} = \alpha x_t + \left(1 - \frac{1}{\mu}\right)\lambda\alpha \ .$$

Substituting (15) into this equation, we obtain a one-dimensional dynamic equation in x_t:

$$\begin{aligned} x_t &= \psi\left(x_{t-1}\right) \\ &= \frac{1}{\alpha}\left(f'(a)\frac{x_{t-1}}{1-\lambda-x_{t-1}} - \rho\sigma^2 x_{t-1} + q\frac{f(a)}{a} - \left(1-\frac{1}{\mu}\right)\lambda\alpha\right) \ . \end{aligned} \tag{16}$$

The graph of the function ψ is illustrated in Figure 1.

Notice that $e_t \geq 0$ requires that $x_t \leq 1 - \lambda$. Figure 1 also shows two steady states $\overline{x}_1 < \overline{x}_2$. Because of (15), the associated growth rates are $\overline{\gamma}_1 > \overline{\gamma}_2$. The steady state with the higher growth rate (with the lower bond share) is asymptotically stable and the other steady state is unstable.[7] Both steady states have

[7]Notice that in our model there in no indeterminacy since the initial bond share x_0 does not depend on expectations. This is a consequence of our assumption that neither savings nor the coefficient of the Clower constraint λ are influenced by inflation expectations. Compare also with Schreft and Smith (1997, p. 175).

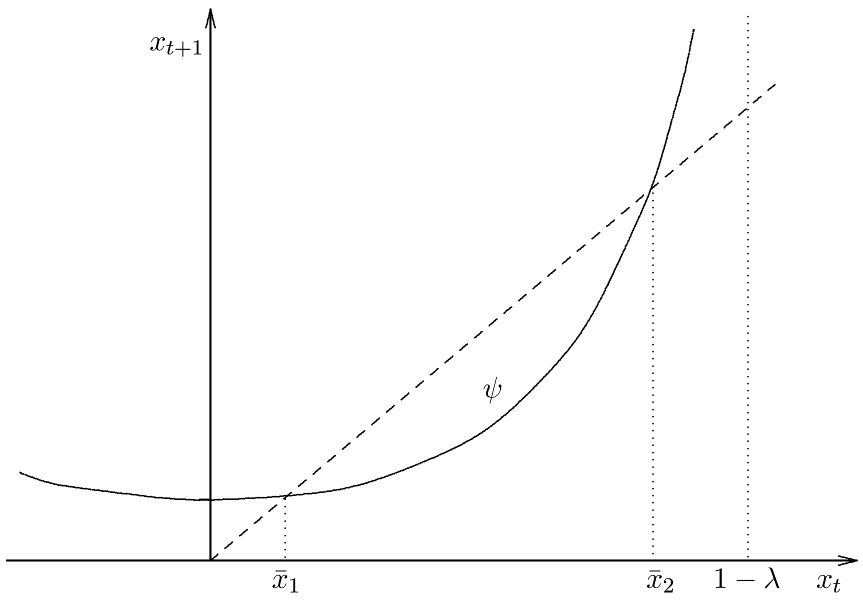

Figure 1: Steady states in the case of constant money growth.

a positive level of government debt provided that the equation $\psi(x) - \psi(0) = x$ has a positive solution and that $\psi(0) > 0$. These conditions are fulfilled if

$$(1-\lambda)\left(\alpha + \rho\sigma^2\right) > f'(a) \quad \text{and} \quad q > \left(1 - \frac{1}{\mu}\right)\lambda\left(1 - \frac{f'(a)a}{f(a)}\right) .$$

For instance, there are two positive steady states if the uncertainty (or risk aversion) and the fiscal share are not too low. A higher level of government spending or a lower money growth rate shift the graph of ψ upwards, increase $\overline{x}_1$ and decrease $\overline{\gamma}_1$. Thus, fiscal policy affects growth negatively, but money growth has a positive impact on growth. Because of (10), the bond return increases and the equity premium falls which shifts savings from equity to bonds and reduces growth. A further consequence is that the equity premium and the growth rate are positively related. Notice, however, that the comparative statics effects on the other (unstable) steady state are opposite.

It remains to check whether the condition $R_t^B > R_t^M$ is satisfied in the steady state. Using (10) and (12), this condition is satisfied in a steady state if

$$f'(a) - \frac{\rho\sigma^2}{\alpha}\overline{\gamma} > \frac{1}{\mu}\overline{\gamma} \; . \tag{17}$$

If the government debt is positive in the stable steady state, we have $\overline{\gamma}_2 < \overline{\gamma}_1 < (1-\lambda)\alpha$, and therefore (17) holds in both steady states whenever (17) is satisfied for $\overline{\gamma} = (1-\lambda)\alpha$. If the production function is Cobb-Douglas, $f(a) = ca^\nu$, a sufficient condition for (17) to hold in both steady states is that

$$\frac{f(a)}{a}\left(\nu\frac{1}{1-\lambda}+(\nu-1)\frac{1}{\mu}\right)>\rho\sigma^2\ .$$

Necessary for this condition to be fulfilled is that the term in the brackets is positive which is the case when the money growth rate is larger than $(1-\lambda)(1-\nu)/\nu$ and which is for sure satisfied when $\nu \geq 1/2$.

Finally, we consider the situation of a zero nominal interest rate in which consumers are indifferent between holding money and bonds. Unlike the model of Hahn and Solow (1995, Chapter 2), our economy has generically no steady state with that feature. To see this, notice that in this case the dynamical system is described by (9), by

$$\frac{m_{t+1}}{\mu m_t}=R_t^M=R_t^B=f'(a)-\frac{\rho\sigma^2}{\alpha}\frac{e_t}{e_{t-1}} \tag{18}$$

and, using (8) and (18), by

$$\frac{m_t}{\mu m_{t-1}}b_{t-1}+q\frac{f(a)}{a}e_{t-1}=b_t+m_t\left(1-\frac{1}{\mu}\right)\ . \tag{19}$$

Again, these three equations are homogenous of degree one in (b, m, e) and they can be reduced to two equations using the variables $\gamma_t = e_t/e_{t-1}$, $\beta_t = b_t/e_t$ and $\nu_t = m_t/e_t$. (18) yields

$$\gamma_{t+1}\frac{\nu_{t+1}}{\mu\nu_t}=f'(a)-\frac{\rho\sigma^2}{\alpha}\gamma_t\ , \tag{20}$$

(19) becomes

$$\gamma_t\frac{\nu_t}{\mu\nu_{t-1}}\beta_{t-1}+q\frac{f(a)}{a}=\beta_t\gamma_t+\nu_t\gamma_t\left(1-\frac{1}{\mu}\right)\ , \tag{21}$$

and (9) is

$$\nu_t+\beta_t+1=\frac{\alpha}{\gamma_t}\ . \tag{22}$$

(20) defines a unique stationary growth rate by

$$\overline{\gamma}=f'(a)\left(\frac{1}{\mu}+\frac{\rho\sigma^2}{\alpha}\right)^{-1}\ .$$

(21) implies that a steady state has to fulfill

$$q\frac{f(a)}{a}=\overline{\gamma}\left(1-\frac{1}{\mu}\right)(\overline{\beta}+\overline{\nu})\ .$$

But this latter condition is in general not compatible with (22) since this would require that

$$q\frac{f(a)}{a} = \left(1 - \frac{1}{\mu}\right)(\alpha - \overline{\gamma}) \ ,$$

which can only hold true for very particular parameter constallations (for instance if $q = 0$ and $\mu = 1$ as in the model of Hahn and Solow (1995), but not under more general policy specifications).

The results of this section are summarized as follows.

Proposition 2 *Under constant money growth, there are no or two steady states in which consumers are liquidity constrained. The steady state with the higher growth rate is asymptotically stable, and the other steady state is unstable. The growth rate and the equity premium at the stable steady state are higher if money growth is faster or if the fiscal share is lower. An equilibrium with portfolio indifference generally does not exist.*

3.2 Fixed Money–bond Ratio

We assume here along the lines of Schreft and Smith (1998) that the monetary authority stabilizes the money supply relative to the level of public debt in the economy. That is, it fixes the ratio of bonds to money, $b_t/m_t = \kappa$ for all t. Higher levels of κ represent an increase in the bond–money ratio and correspond to a tighter monetary regime. Starting with the case $R_t^B > R_t^M$, $m_t = \lambda\alpha e_{t-1}$ and (9) yield

$$e_t = (1 - (1+\kappa)\lambda)\,\alpha e_{t-1}.$$

Therefore

$$\gamma_t = (1 - (1+\kappa)\lambda)\,\alpha = \overline{\gamma} \tag{23}$$

which means that the growth rate is constant and independent of q and $\rho\sigma^2$. Since the central bank fixes the mix of government liabilities (bonds and outside money) over time, fiscal policy has no effect on growth. Furthermore it follows that

$$x_t = \kappa\lambda = \overline{x}$$

and thus also x_t is constant and independent of q and $\rho\sigma^2$. By (10) the steady-state bond return is

$$R^B = f'(a) - \frac{\rho\sigma^2}{\alpha}\overline{\gamma} \ . \tag{24}$$

These results imply that a tighter monetary policy (an increase in κ) increases bond supply and the bond return, reduces the equity premium, increases the ration of bonds to capital, and decreases the growth rate. From (8), (10) and $m_t = \lambda\alpha e_{t-1}$

$$R_{t-1}^M = (1+\kappa)\,\gamma_{t-1} - \left(f'(a) - \frac{\rho\sigma^2}{\alpha}\gamma_{t-1}\right)\kappa - \frac{q}{\lambda\alpha}\frac{f(a)}{a}\gamma_{t-1}$$

which in equilibrium becomes

$$R^{M}=\left(1+\kappa+\frac{\rho\sigma^{2}}{\alpha}\kappa-\frac{q}{\lambda\alpha}\frac{f(a)}{a}\right)\overline{\gamma}-f'(a)\,\kappa. \tag{25}$$

This enables us now to check the validity of the assumption that $R^{M}<R^{B}$. Using (23), (24) and (25) we can restate this condition as

$$\left((1+\kappa)\left(1+\frac{\rho\sigma^{2}}{\alpha}\right)-\frac{q}{\lambda\alpha}\frac{f(a)}{a}\right)(1-(1+\kappa)\,\lambda)\,\alpha<(1+\kappa)\,f'(a)\,.$$

For given values of the other parameters, this condition can always be fulfilled by assuming κ and/or q big enough. Moreover, in the special case $\rho\sigma^{2}=q=0$ and $f(a)=ca^{\nu}$ it becomes

$$1+\kappa>\frac{1}{\lambda}\left(1-\frac{\nu}{1-\nu}\right)$$

which is for sure satisfied when $\nu\geq 1/2$.[8]

To complete the analysis we also consider the case of portfolio indifference, i.e. $R_{t}^{M}=R_{t}^{B}$. From (9) we obtain

$$(1+\kappa)\,m_{t}+e_{t}=\alpha e_{t-1}$$

whereas (8) and (10) yield

$$\left(f'(a)-\frac{\rho\sigma^{2}}{\alpha}\frac{e_{t-1}}{e_{t-2}}\right)(1+\kappa)\,m_{t-1}+q\frac{f(a)}{a}e_{t-1}=(1+\kappa)\,m_{t}.$$

Setting as before $\gamma_{t}=e_{t}/e_{t-1}$ and $\nu_{t}=m_{t}/e_{t}$ these equations can be written as

$$(1+\kappa)\,\nu_{t}\gamma_{t}+\gamma_{t}=\alpha$$

and

$$\left(f'(a)-\frac{\rho\sigma^{2}}{\alpha}\gamma_{t-1}\right)(1+\kappa)\,\nu_{t-1}+q\frac{f(a)}{a}=(1+\kappa)\,\nu_{t}\gamma_{t}.$$

Solving for

$$\gamma_{t}=\frac{\alpha}{1+\chi_{t}}$$

with $\chi_{t}=(1+\kappa)\,\nu_{t}$ from the first equation and inserting in the second yields

$$\left(f'(a)-\frac{\rho\sigma^{2}}{1+\chi_{t-1}}\right)\chi_{t-1}+q\frac{f(a)}{a}=\frac{\alpha\chi_{t}}{1+\chi_{t}}.$$

[8]When $\rho\sigma^{2}=0$, the consumer is risk neutral or there is no uncertainty. This can be considered a limiting case of our setting in which, from (7) and (10), $\overline{R}_{t}^{E}=R_{t}^{B}$, and the consumer's problem can then be written $\max_{m_{t},b_{t},e_{t}}R_{t}^{M}m_{t}+R_{t}^{B}\,(b_{t}+e_{t})$ s.t. $m_{t}+b_{t}+e_{t}\leq w_{t},\ m_{t}\geq\lambda w_{t},\ e_{t}\geq 0$.

Finally solving for χ_t we obtain

$$\chi_t = \frac{\left(f'(a) - \frac{\rho\sigma^2}{1+\chi_{t-1}}\right)\chi_{t-1} + q\frac{f(a)}{a}}{\alpha - \left(f'(a) - \frac{\rho\sigma^2}{1+\chi_{t-1}}\right)\chi_{t-1} - q\frac{f(a)}{a}} = \phi\left(\chi_{t-1}\right).$$

To explore the existence of steady states let us start with the case $\rho\sigma^2 = q = 0$. Then $\overline{\chi} = 0$ is one steady state and, since $\phi(\chi) \to \infty$ as $\chi \to \alpha/f'(a)$, an additional positive (unstable) steady state exists if $\phi'(0) < 1$. In the special case $f(a) = ca^\nu$ this means $\nu < 1/2$. Then, as q is slightly increased, $\phi(0) > 0$ and a stable positive steady state $\overline{\chi}$ emerges. This remains true when $\rho\sigma^2$ is slightly increased, too. However, this steady state is only an equilibrium of our model if also the liquidity constraint $m_t \geq \lambda\alpha e_{t-1}$ is satisfied. This constraint means that

$$\overline{\chi} \geq \frac{\lambda(1+\kappa)}{1-\lambda(1+\kappa)},$$

which can be satisfied, if at all, only when κ is small enough. That is, only a sufficiently loose monetary policy may give rise to a steady state in which the nominal interest rate is zero and in which consumers are portfolio indifferent. In any way, whenever such a steady state exists, it is clear that a variation in κ does not change $\overline{\chi}$ and therefore does not change $\overline{\gamma}$ either. In conclusion we thus have:

Proposition 3 *Under a fixed bond–money ratio $b_t/m_t = \kappa$, there is at most one equilibrium in which consumers are liquidity constrained and which must be a steady state. In such an equilibrium a tightening of the monetary regime (i.e. a higher κ) decreases the growth rate, while fiscal policy does not affect growth. If the monetary regime is sufficiently loose, steady states in which consumers are portfolio indifferent may also exist, but in these steady states the growth rate is independent of κ.*

3.3 Interest Rate Targeting

In this case the central bank intends to fix the nominal interest factor I_t at a value $I > 1$ for all t. Since $I = R_t^B/R_t^M$, consumers are liquidity constrained, $m_t = \lambda\alpha e_{t-1}$. Equation (8) now becomes

$$\left(f'(a) - \frac{\rho\sigma^2}{\alpha}\gamma_{t-1}\right)b_{t-1} + q\frac{f(a)}{a}e_{t-1} = b_t + \lambda\alpha e_{t-1} - \frac{1}{I}\left(f'(a) - \frac{\rho\sigma^2}{\alpha}\gamma_{t-1}\right)\lambda\alpha e_{t-2}.$$

Setting again $x_t = b_t/(\alpha e_{t-1})$, dividing by e_{t-1} and using (15) yields

$$\left(f'(a) - \rho\sigma^2(1-\lambda-x_{t-1})\right)\frac{x_{t-1}}{1-\lambda-x_{t-1}} + q\frac{f(a)}{a}$$

$$= \alpha x_t + \lambda\alpha - \frac{1}{I}\left(\frac{f'(a)\lambda}{1-\lambda-x_{t-1}} - \rho\sigma^2\lambda\right).$$

Solving for x_t we obtain

$$x_t = \frac{1}{\alpha}\left(f'(a)\frac{x_{t-1}+\lambda/I}{1-\lambda-x_{t-1}} - \rho\sigma^2 x_{t-1} + q\frac{f(a)}{a} - \lambda\left(\alpha + \frac{\rho\sigma^2}{I}\right)\right).$$

The graph of this function is qualitatively the same as the one shown in Figure 1. Both steady states have a positive level of government debt if

$$(1-\lambda)\left(\alpha+\rho\sigma^2\right) > f'(a)\left(1+\frac{\lambda}{(1-\lambda)\,I}\right) \text{ and } q > \frac{\lambda a}{f(a)}\left(\alpha + \frac{\rho\sigma^2}{I} - \frac{f'(a)}{(1-\lambda)\,I}\right)$$

hold. This is for example true when the uncertainty/risk aversion and the fiscal share are high enough. Moreover, higher uncertainty or risk aversion and a lower level of government spending shift the curve downwards decreasing $\overline{x}_1$ and increasing $\overline{\gamma}_1$. Regarding the effect of a change in the nominal interest rate, it can be obtained from

$$\frac{\partial x_t}{\partial I} = \frac{\lambda}{\alpha}\left(\rho\sigma^2 - \frac{f'(a)}{1-\lambda-x_{t-1}}\right)\frac{1}{I^2}.$$

For small uncertainty/risk aversion this derivative is negative implying that an increase in I decreases $\overline{x}_1$ and increases $\overline{\gamma}_1$ whereas for large $\rho\sigma^2$ the effect is reversed. If the uncertainty and risk aversion are low, the increase in the nominal interest rate is accompanied by a larger increase in the inflation rate which lowers the real interest rate and raises capital investment. On the other hand, if uncertainty and risk aversion are large, the inflation rate increases less than the nominal interest rate, and so the real interest rate increases as well which affects growth negatively. We have therefore obtained the following result.

Proposition 4 *Under interest rate targeting there exist no or two steady states. The steady state with the higher growth rate is asymptotically stable. A decrease in the fiscal share increases the growth rate and the risk premium. The effect of the nominal interest target on growth is positive when uncertainty or risk aversion is small whereas it is negative for uncertainty and risk aversion large.*

3.4 Inflation Targeting

Under inflation targeting the central bank aims to have $R_t^M = R^M$ for all t and $R^M > 0$ predetermined. Proceeding as in the previous section, the case $R_t^B > R^M$ yields

$$\left(f'(a) - \rho\sigma^2(1-\lambda-x_{t-1})\right)\frac{x_{t-1}}{1-\lambda-x_{t-1}} + q\frac{f(a)}{a}$$

$$= \alpha x_t + \lambda\alpha \left(1 - \frac{R^M}{\alpha(1-\lambda-x_{t-1})}\right)$$

and therefore

$$x_t = \frac{1}{\alpha}\left(\frac{f'(a)x_{t-1} + \lambda R^M}{1-\lambda-x_{t-1}} - \rho\sigma^2 x_{t-1} + q\frac{f(a)}{a} - \lambda\alpha\right).$$

The graph of this function is again as in Figure 1. Both steady states involve positive government debt if

$$(1-\lambda)\left(\alpha + \rho\sigma^2\right) > f'(a) + \frac{\lambda R^M}{1-\lambda} \quad \text{and} \quad q > \frac{\lambda a}{f(a)}\left(\alpha - \frac{R^M}{1-\lambda}\right)$$

hold. As before, these conditions can be fulfilled if uncertainty, risk aversion and government consumption are large enough. Regarding the comparative statics properties of the steady states, they are analogous to the case of interest rate targeting. The only difference is that now an increase in the inflation rate (a lower R^M) unambiguously lowers the real interest rate and increases the growth rate.

When is it true that in a steady state $R^B > R^M$? From (10) a necessary condition is $f'(a) > R^M$. This means that too low an inflation rate is unsustainable on a steady-state growth path with liquidity-constrained consumers. Portfolio indifference, on the other hand, requires from (10) for the growth rate to be constant with value

$$\gamma = \left(f'(a) - R^M\right)\frac{\alpha}{\rho\sigma^2} \tag{26}$$

which cannot be fulfilled when $\rho\sigma^2 = 0$ (unless R^M happens to be equal to $f'(a)$). However, also when $\rho\sigma^2$ is positive, steady states with $R^B = R^M$ in general do not exist. To see this, set as earlier $\beta_t = b_t/e_t$ and $\nu_t = m_t/e_t$. Then (9) yields

$$\nu_t = \frac{\alpha}{\gamma} - 1 - \beta_t\ , \tag{27}$$

whereas (8) implies

$$q\frac{f(a)}{a} = (\beta_t + \nu_t)\,\gamma - R^M(\nu_{t-1} + \beta_{t-1}).$$

Inserting for ν_t we obtain

$$q\frac{f(a)}{a} = \alpha - \gamma - R^M\left(\frac{\alpha}{\gamma} - 1\right)$$

which shows that an equilibrium can only exist if R^M fulfils this equation. But under inflation targeting R^M is a predetermined magnitude, and thus a steady state with portfolio indifference generally does not exist.

We summarize the results on inflation targeting as follows.

Proposition 5 *Under inflation targeting there are no or two steady states with liquidity-constrained consumers. The steady state with the higher growth rate is asymptotically stable. A higher inflation target or a lower fiscal share increase the growth rate and the risk premium. Equilibria with portfolio indifference generally do not exist.*

Notice that the results of Sections 3.3 and 3.4 are compatible with those of 3.1 and 3.2. Indeed, denoting with $\overline{\mu}$ the steady-state increase in the nominal money stock, it is related to inflation by $\overline{\mu} = \overline{\gamma}/R^M$. An increase in the inflation rate increases $\overline{\gamma}$ and hence $\overline{\mu}$. Thus $\overline{\mu}$ and $\overline{\gamma}$ are correlated positively, as was predicted in the case of constant money growth. Regarding the ratio of bonds and and the money stock $\overline{\kappa} = \overline{b}/\overline{m}$, from (14) and $m_t = \lambda\alpha e_{t-1}$ it is equal to $(1 - \lambda - \overline{\gamma}/\alpha)/\lambda$. An increase in the inflation rate increases (in the stable steady state) $\overline{\gamma}$ and hence diminishes $\overline{\kappa}$, meaning that $\overline{\kappa}$ and $\overline{\gamma}$ are inversely correlated as stated in section 3.2.

4 Adaptive Expectations

We have shown in the previous section that our model exhibits in most cases two steady states in which consumers are liquidity constrained, and we argued that the steady state which is stable in the perfect foresight dynamics is the relevant one, whereas the unstable steady state whose policy features are opposite is of minor importance. We give now further support to this argument by showing that a steady state which is stable under perfect foresight is also stable under adaptive expectations, and vice versa. Thus our model predicts the same outcome in the forward perfect foresight dynamics as under learnung with adaptive expectations. This result contrasts to the result of Grandmont and Laroque (1986) who show that, under their assumptions, stability in the backward perfect foresight dynamics relates to stability in the actual learning dynamics.[9] The following analysis is restricted to the case of constant money growth, but we believe that similar results can be obtained with an inflation targeting or an interest rate targeting policy.

Since the distribution of the capital return is the same in every period irrespective of the state of the economy, we assume that the consumer has learned this distribution perfectly and forecasts correctly its mean and variance as given by (7). However, the consumer does not perfectly foresee the inflation rate and we assume that he holds in period t an inflation forecast $R_t^{M,e}$. At the nominal interest rate I_t his forecast of the real interest rate is then $R_t^{B,e} = I_t R_t^{M,e}$. A *temporary equilibrium* in period t, given an inflation expectation $R_t^{M,e}$, is a

[9] In Grandmont and Laroque (1986) the law of motion of the economic system is of the type $X_t = F\left(X_{t+1}^e\right)$ whereas our system is of the form $X_t = F\left(X_{t-1}, X_t^e\right)$. Expectations of the future state X_{t+1}^e do not enter our temporary equilibrium map, cf. footnote 7.

real wage w_t, a nominal interest rate I_t, and an inflation factor R^M_{t-1} (or equivalently, a price level p_t) such that the labor market and all financial markets are in equilibrium. By Walras's law, the goods market is then in equilibrium, too. Labor market equilibrium follows again from (6), whereas the equity market is (by construction) in equilibrium whenever consumers forecast the distribution of the equity return correctly. Hence, there remains to consider the money and the bond market.

When consumers are liquidity constrained ($I_t > 1$) and when investment (= equity demand) is positive ($f'(a) - I_t R^{M,e}_t > 0$), money and bond demand are given by

$$m^d_t = \lambda w_t = \lambda \alpha e_{t-1} \qquad \text{and} \qquad b^d_t = (1-\lambda)\alpha e_{t-1} - \frac{f'(a) - I_t R^{M,e}_t}{\rho\sigma^2} \alpha e_{t-1} \ .$$

Since nominal money grows at constant rate μ, money and bond supply are

$$m^s_t = \mu R^M_{t-1} m_{t-1} \qquad \text{and} \qquad b^s_t = I_{t-1} R^M_{t-1} b_{t-1} + q\frac{f(a)}{a} e_{t-1} + R^M_{t-1} m_{t-1}(1-\mu) \ .$$

Equilibrium in the money market implies

$$m_t = \lambda \alpha e_{t-1} \quad \text{and} \quad R^M_{t-1} = \frac{\lambda \alpha e_{t-1}}{\mu m_{t-1}} \quad , \tag{28}$$

whereas the bond market is in equilibrium if

$$\begin{aligned} b_t &= I_{t-1} R^M_{t-1} b_{t-1} + q\frac{f(a)}{a} e_{t-1} + R^M_{t-1} m_{t-1}(1-\mu) \ , & (29) \\ I_t &= \frac{1}{R^{M,e}_t}\left(f'(a) - \rho\sigma^2(1 - \lambda - \frac{b_t}{\alpha e_{t-1}})\right) > 1 \ . & (30) \end{aligned}$$

Again using x_t for the bond share, substituting (15) for the growth rate, and substituting I_{t-1} in (29) by (30), (28) and (29) yield

$$R^M_{t-1} = g(x_{t-1}) = \frac{\alpha}{\mu}(1 - \lambda - x_{t-1}) \quad ,$$

$$\begin{aligned} x_t &= \Psi(x_{t-1}, R^{M,e}_{t-1}) \\ &= \frac{1}{\mu R^{M,e}_{t-1}} \left(f'(a) - \rho\sigma^2(1 - \lambda - x_{t-1})\right) x_{t-1} + \frac{q}{\alpha}\frac{f(a)}{a} + \lambda\left(\frac{1}{\mu} - 1\right) \ . \end{aligned}$$

It is immediate from these definitions that the functions Ψ and g satisfy[10]

$$\Psi_1 > 0 \ , \ \ \Psi_2 < 0 \ , \ \ g' < 0 \ . \tag{31}$$

[10] Ψ_i denotes the ith partial derivative of Ψ. The first inequality follows since $f'(a) - \rho\sigma^2(1 - \lambda - x_{t-1}) > 0$ whenever $I_t > 0$.

Notice that the assumption of perfect foresight, $R_{t-1}^{M,e} = R_{t-1}^{M} = g(x_{t-1})$, yields the perfect foresight dynamics

$$x_t = \Psi(x_{t-1}, g(x_{t-1})) = \psi(x_{t-1}) \tag{32}$$

as derived in Section 3.1. Assuming adaptive expectations, the actual dynamics is described by

$$x_t = \Psi(x_{t-1}, R_{t-1}^{M,e}) \quad \text{and} \quad R_t^{M,e} = \eta R_{t-1}^{M,e} + (1-\eta) g(x_{t-1}) \tag{33}$$

with some adjustment parameter $\eta \in [0, 1)$. The stability features of the forward perfect foresight dynamics turn out to be equivalent to those under adaptive expectations, provided that (31) holds. As a consequence of this result, the high growth steady state $\overline{x}_1$ of Section 3.1 is also stable under adaptive expectations and the low growth steady state $\overline{x}_2$ is unstable under adaptive expectations.

Proposition 6 *Suppose that (31) holds and that $\overline{x}$ is a steady state with $|\psi'(\overline{x})| \neq 1$. Then $\overline{x}$ is locally stable under (32) if and only if $(\overline{x}, g(\overline{x}))$ is locally stable under (33).*

Proof: Suppose first that $\overline{x}$ is locally stable under (32). Therefore

$$\Psi_1 + \Psi_2 g' < 1 \tag{34}$$

holds at $\overline{x}$. Let J denote the Jacobian of (33) evaluated at $(\overline{x}, g(\overline{x}))$. Then $\det J = \eta(\Psi_1 + \Psi_2 g') - \Psi_2 g'$ and $\operatorname{Tr} J = \Psi_1 + \eta$. The steady state $(\overline{x}, g(\overline{x}))$ is locally stable under (33) if the conditions $\det J < 1$, $\det J > \operatorname{Tr} J - 1$ and $\det J > -\operatorname{Tr} J - 1$ are satisfied. The first of these conditions means that

$$\eta\,(\Psi_1 + \Psi_2 g') < 1 + \Psi_2 g'$$

which is clearly fulfilled because of (34) and $\Psi_2 g' > 0$. The second condition is equivalent to (34) and is thus satisfied. The third condition is fulfilled if

$$(1+\eta)\,(1+\Psi_1) > (1-\eta)\,\Psi_2 g'$$

and is satisfied since $\Psi_1 > 0$ and $\Psi_2 g' < \Psi_1 + \Psi_2 g' < 1$. Hence $(\overline{x}, g(\overline{x}))$ is locally stable under (33).

Suppose conversely that $(\overline{x}, g(\overline{x}))$ is locally stable under (33) and that $\overline{x}$ is unstable under (32). Thus, $\Psi_1 + \Psi_2 g' > 1$, which implies that $\det J < \operatorname{Tr} J - 1$. But this implies that one eigenvalue of J has modulus greater than one, a contradiction. $\square$

5 Conclusions

We have considered an economy where consumers hold two outside assets (money and government bonds) and capital, and where the central bank can apply different monetary strategies to promote growth. Multiple steady states with positive nominal interest rates exist, whereas portfolio indifference steady states in which the nominal interest rate is zero exist only in pathological situations. This result contrasts with the analysis of Hahn and Solow (1995, Chapter 2) who focus on portfolio indifference steady states and who argue that they give rise to instability and endogenous fluctuations.

However, even though there are multiple steady states, only one of them is stable, not only in the dynamics with perfect foresight but also in the dynamics with adaptive expectations. An expansive monetary policy enhances growth in the unique stable steady state, and this result is irrespective of the monetary strategy. Only under an interest rate targeting policy, the outcome depends crucially on the policy's effect on the real interest rate. Typically, however, an expansive monetary policy lowers the real interest rate, raises the risk premium and promotes capital accumulation.

To keep our model analytically tractable, we have imposed some simplifying assumptions whose relaxation would be worthful to investigate. We assumed that consumers consume only in old age and save all their labor income. A more general savings behaviour would allow to study the interaction between portfolio choice and savings, and this interaction can be expected to be relevant also for the policy conclusions. Another simplification is the assumption of an Arrow–Romer externality leading to endogenous growth, and it would be interesting to examine other endogenous growth models in which growth is generated from human capital formation or from innovations.

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